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Origin of FRW cosmology in slow-roll inflation from non-compact Kaluza–Klein theory

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Abstract. Using a recently introduced formalism we discuss slow-roll inflation from Kaluza–Klein theory without the cylinder condition. In particular, some examples corresponding to polynomic and hyperbolic ϕ-potentials are studied. We find that the evolution of the fifth coordinate should be determinant for both the evolution of the early inflationary universe and the quantum fluctuations.

1 Introduction

The possibility that our universe is embedded in a higher dimensional space has generated a great deal of active interest. In brane-world (BW) [1–3] and space-time-matter (STM) [4] theories the usual constraint on Kaluza–Klein (KK) models, namely the cylinder condition, is relaxed so the extra dimensions are not restricted to be compact. An alternative idea pertaining to geometrical compactification has been explored by Randall and Sundrum [5], who demonstrated that for sufficiently low energies the probability of losing energy to the KK states is very small. The first important question concerning solutions in 5D is to check whether they give back the standard 4D results. In particular, cosmological models should be developed from a Friedmann–Robertson–Walker (FRW) cosmology.

Inflation has nowadays become a standard ingredient for the description of the early universe. In fact, it solves some of the problems of the standard big-bang scenario and also makes predictions about cosmic microwave background radiation (CMBR) anisotropies which are being measured with higher and higher precision. The first model of inflation was proposed by Starobinsky in 1979 [6]. A much simpler inflationary model with a clear motivation was developed by Guth in the 80's [7], in order to solve some of the shortcomings of the big-bang theory, and in particular, to explain the extraordinary homogeneity of the observable universe. However, the universe after inflation in this scenario becomes very inhomogeneous. Following a detailed investigation of this problem, Guth and Weinberg concluded that the old inflationary model could not be improved [8]. These problems were sorted out by Linde in 1983 with the introduction of chaotic inflation [9]. In this scenario inflation can occur in theories with potentials such as $V(\phi) \sim \phi^n$. It may begin in the absence of thermal equilibrium in the early universe, and it may start even at the Planck density, in which case the problem of initial conditions for inflation can be easily solved [10].

Recently there has been significant progress made in the field of string cosmology [11,12]. Arguably, among the greatest triumphs are the string realizations of the inflationary universe paradigm [13]. Moreover, inflationary cosmology from STM models has been a subject of great interest in the last years [14–16]. In a novel approach recently developed [17] it was suggested that the evolution of the early universe could be described by a geodesic trajectory of a 5D metric, so that the effective 4D FRW background metric should be a hypersurface on a constant fifth dimension. In this paper we extend this approach to other inflationary models.

This work is organized as follows: in Sect. 2 the formalism proposed in [17] is reviewed and extended. In Sect. 3 we study inflationary dynamics taking into account the slow-roll conditions. Section 4 discusses inflationary models coming from polynomic (quadratic and quartic) and hyperbolic ϕ-potentials. Finally, in Sect. 5 we develop some final comments.

2 Formalism reviewed and extended

We consider the 5D metric, recently introduced by Ledesma and Bellini (LB) [17]

$$\mathrm{d}S^2 = \psi^2 \mathrm{d}N^2 - \psi^2 \mathrm{e}^{2N} \mathrm{d}r^2 - \mathrm{d}\psi^2 \,, \qquad (1)$$

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where the parameters (N, r) are dimensionless, and the fifth coordinate ψ has spatial unities. The metric (1) describes a flat 5D manifold in apparent vacuum ($G_{AB} = 0$). Furthermore, on hypersurfaces $\psi = $ const. the 4D induced pressure ($\mathbf{p}$) and energy density (ρ) are given by

$$\mathbf{p} = -\frac{3}{8\pi G\psi^2}\,, \qquad \rho = \frac{3}{8\pi G\psi^2}\,, \tag{2}$$

such that the 4D equation of state on hypersurfaces with constant ψ is $\mathbf{p} = -\rho$, which corresponds to a vacuum. Hence, systems in an apparent vacuum on hypersurfaces with constant ψ in a 5D manifold described by (1) comply with a 4D vacuum equation of state. In particular, the case for which the squared Hubble parameter is given by $H_0^2 = \Lambda/(3\psi^2)$ represents a de Sitter expansion [19] governed by the cosmological constant Λ.

As in [17] we shall consider the case where N only depends on the cosmic time t: $N = N(t)$. The relevant Christoffel symbols for the geodesic of the metric (1) in a comoving frame $U^r = \partial r/\partial S = 0$ are

$$\Gamma^N_{\psi\psi} = 0\,, \quad \Gamma^N_{\psi N} = 1/\psi\,, \quad \Gamma^\psi_{NN} = \psi\,, \quad \Gamma^\psi_{N\psi} = 0\,, \tag{3}$$

so that the geodesic dynamics $\frac{\mathrm{d}U^C}{\mathrm{d}S} = \Gamma^C_{AB}U^AU^B$ is described by the following equations of motion for the velocities U^A:

$$\frac{\mathrm{d}U^N}{\mathrm{d}S} = -\frac{2}{\psi}U^NU^\psi\,, \tag{4}$$

$$\frac{\mathrm{d}U^\psi}{\mathrm{d}S} = -\psi U^NU^N\,, \tag{5}$$

$$\psi^2U^NU^N - U^\psi U^\psi = 1\,, \tag{6}$$

where (6) describes the constraint condition $g_{AB}U^AU^B = 1$. From the general solution $\psi U^N = \cosh[S(N)]$, $U^\psi = -\sinh[S(N)]$, we obtain the equation that describes the geodesic evolution for ψ:

$$\frac{\mathrm{d}\psi}{\mathrm{d}N} = \frac{U^\psi}{U^N} = -\psi\tanh[S(N)]\,, \tag{7}$$

where $S(N) = -N$ gives the number of e-folds of the universe. If we take $\tanh[S(N)] = -1/u(N)$, we obtain

$$\psi(N) = \psi_0 \mathrm{e}^{\int \mathrm{d}N/u(N)}\,, \tag{8}$$

for the velocities

$$U^\psi = -\frac{1}{\sqrt{u^2(N)-1}}\,, \qquad U^N = \frac{u(N)}{\psi\sqrt{u^2(N)-1}}\,, \tag{9}$$

where ψ_0 in (8) is a constant of integration. As in a previous paper [17], we are interested in the case in which $\psi = H^{-1}$, where H is the classical Hubble parameter. With this choice the constant $\psi_0 = H_0^{-1}$ describes the primordial Hubble horizon, which should be of the order of the Planck length. Furthermore, the function u is given by $u(N) = -\frac{H}{\mathrm{d}H/\mathrm{d}N} > 0$, because $\mathrm{d}H/\mathrm{d}N < 0$ during inflation.

The resulting 5D metric is given by

$$\mathrm{d}S^2 = \mathrm{d}t^2 - \mathrm{e}^{2\int H(t)\mathrm{d}t}\mathrm{d}R^2 - \mathrm{d}L^2\,, \tag{10}$$

with $t = \int \psi(N)\mathrm{d}N$, $R = r\psi$ and $L = \psi_0$. With this representation, we obtain the following new velocities $\hat{U}^A = \frac{\partial \hat{x}^A}{\partial x^B}U^B$:

$$U^t = \frac{2u(t)}{\sqrt{u^2(t)-1}}\,, \qquad U^R = \frac{-2r}{\sqrt{u^2(t)-1}}\,, \qquad U^L = 0\,, \tag{11}$$

where the old velocities U^B are U^N, $U^r = 0$ and U^ψ. Furthermore, the velocities $\hat{U}^B$ comply with the constraint condition

$$\hat{g}_{AB}\hat{U}^A\hat{U}^B = 1\,. \tag{12}$$

The important fact here is that the new frame gives us an effective spatially flat FRW metric embedded in a 5D manifold where the fifth coordinate $L = \psi_0$ is the primordial Hubble horizon, which emerges naturally as a constant in this representation.

The solution $N = \mathrm{arctanh}[1/u(t)]$ corresponds to a power-law expanding universe with time dependent power $p(t)$ for a scale factor $a \sim t^{p(t)}$. Since $H(t) = \dot{a}/a$, the resulting Hubble parameter is

$$H(t) = \dot{p}\ln(t/t_0) + p(t)/t, \tag{13}$$

where t_0 is the initial time. The function u written as a function of time is

$$u(t) = -\frac{H^2}{\dot{H}}\,, \tag{14}$$

where the dot represents the derivative with respect to the time.

With this representation the universe can be viewed as born in a state with $S \simeq 0$ (i.e., in a 4D vacuum state $\mathbf{p} \simeq -\rho$), where the fifth coordinate is given by the Hubble horizon in a comoving frame $\mathrm{d}r = 0$, such that the effective 4D spacetime is a FRW metric

$$\mathrm{d}S^2 = \mathrm{d}t^2 - \mathrm{e}^{2\int H(t)\mathrm{d}t}\mathrm{d}R^2 - \mathrm{d}L^2 \to$$
$$\mathrm{d}s^2 = \mathrm{d}t^2 - \mathrm{e}^{2\int H(t)\mathrm{d}t}\mathrm{d}R^2\,. \tag{15}$$

In this framework we can define the $5D$ lagrangian

$$\mathcal{L}(\phi, \phi_{,A}) = -\sqrt{-^{(5)}g}\left[\frac{1}{2}g^{AB}\phi_{,A}\phi_{,B} + V(\phi)\right]\,, \tag{16}$$

for the scalar field $\phi(N, r, \psi)$ with the metric (1). Here, $^{(5)}g$ is the determinant of the 5D metric tensor in (1) and $V(\phi)$ is the potential. On the geodesic $N = \mathrm{arctanh}[1/u(t)]$ in the comoving frame $\mathrm{d}r = 0$, the effective 4D lagrangian for the metric (15) is

$$\mathcal{L}\left(\phi, \phi_{,A}\right) \to \tag{17}$$
$$\mathcal{L}\left(\phi, \phi_{,\mu}\right) = -\sqrt{-^{(4)}g}\left[\frac{1}{2}g^{\mu\nu}\phi_{,\mu}\phi_{,\nu} + V(\phi)\right]\,,$$

where ${}^{(4)}g$ is the determinant of the metric tensor in the 4D effective FRW background metric (15) and $\phi(t,R,L) \equiv \phi(t,R)$. In this frame, the 4D energy density and the pressure are [17]

$$8\pi G\rho = 3H^2\,, \tag{18}$$

$$8\pi G\mathbf{p} = -(3H^2 + 2\dot{H})\,, \tag{19}$$

with $H(t) = \dot{a}/a$ for a given scale factor $a(t) \sim t^{p(t)}$ and $L = \psi_0$ is of the order of the Planck length. As was emphasized in a previous work [17] such a formalism can be successfully applied to many inflationary models. With the aim to illustrate this, in the following section we shall study some inflationary models. $V(\phi) \sim \phi^n$.

3 Slow-roll inflation

The dynamics of the inflaton field during inflation is characterized by the equations

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0\,, \tag{20}$$

$$\dot{\phi} = -\frac{M_\mathrm{P}^2}{4\pi} H'(\phi)\,, \tag{21}$$

where the prime denotes the derivative with respect to ϕ. If the slow-roll conditions [18] are fulfilled,

$$\gamma = \frac{M_\mathrm{P}^2}{4\pi}\left(\frac{H'}{H}\right)^2 \ll 1\,, \qquad \eta = \frac{M_\mathrm{P}^2}{4\pi}\frac{H''}{H} \ll 1\,, \tag{22}$$

the Friedmann equation can be approximated by

$$H^2 \simeq \frac{8\pi}{3M_\mathrm{P}^2} V(\phi)\,, \tag{23}$$

such that $H' \simeq \frac{\sqrt{4\pi}}{3\sqrt{2}M_\mathrm{P}}\frac{V'}{V^{1/2}}$, and (21) can be approximated by

$$\dot{\phi} \simeq -\frac{V'}{3H} = -\frac{M_\mathrm{P}}{\sqrt{4\pi}}\frac{V'}{V^{1/2}}\,. \tag{24}$$

In other words, slow-roll conditions imply that

$$\frac{3}{8\pi G\psi^2} \simeq V(\phi)\,, \tag{25}$$

where $G = M_\mathrm{P}^{-2}$ is the gravitational constant and $M_\mathrm{P} = 1.2\ 10^{19}$ GeV is the Planck mass. This is a very important expression that says us that the evolution of the universe [governed by $V(\phi)$] has its origin in the evolution of the fifth dimension $\psi = H^{-1}$. The expression (25) is approximately fulfilled in the early universe before the inflaton field begins to oscillate around the minimum of the potential $V(\phi)$. Note that the function u, written as a function of ϕ, becomes

$$u(\phi) = \frac{4\pi}{M_\mathrm{P}^2}\left(\frac{H}{H'}\right)^2\,, \tag{26}$$

which is exactly the inverse of the slow-roll parameter γ [see the first equation in (22)]: $u = 1/\gamma$. Hence, the condition $u \gg 1$ is guaranteed during inflation. This is a very general result valid for all the models of inflation. Furthermore the condition $u \gg 1$ says us that the velocities (11) are real. They can be written in terms of the Hubble parameter,

$$U^t = \frac{2\left(\frac{H}{H'}\right)^2}{\sqrt{\left(\frac{H}{H'}\right)^4 - \frac{M_\mathrm{P}^4}{16\pi^2}}}\,, \tag{27}$$

$$U^R = -\frac{2r}{\sqrt{16\pi^2\left(\frac{H}{H'M_\mathrm{P}}\right)^4 - 1}}\,, \qquad U^L = 0\,.$$

Furthermore, since $u \gg 1$ during inflation, we can approximate the velocities (27)

$$U^t \simeq 2\,, \qquad U^R \simeq -2r\gamma\,, \qquad U^L = 0\,. \tag{28}$$

The constraint condition (12) implies that (for $\gamma_0 = \frac{\sqrt{3}}{2r}$)

$$\gamma \simeq \gamma_0 \frac{a_0}{a(t)} \ll 1\,, \tag{29}$$

which holds for all $t \geq t_0$ and $r \gg 1$. Note that (29) has its origin in a geometrical property; the condition (12). The constraint condition (29) can be written as

$$u(t)a_0 \simeq u_0 a(t)\,, \tag{30}$$

where $u_0 = r/\sqrt{3} \gg 1$ and $u(t) = -H^2/\dot{H}$.

Potentials like

$$V(\phi) = \frac{\lambda}{n}\phi^n\,, \tag{31}$$

are interesting for inflationary cosmology. For these potentials (24) becomes

$$\dot{\phi} = -\frac{M_\mathrm{P}}{\sqrt{4\pi}}\sqrt{n\lambda}\,\phi^{\frac{n-2}{2}}\,, \tag{32}$$

with solutions

$$\phi^{\frac{4-n}{2}}(t) = \phi_0^{\frac{4-n}{2}} - \frac{M_\mathrm{P}\sqrt{n\lambda}(4-n)}{4\sqrt{\pi}}t, \tag{33}$$

$$\phi(t) = \phi_0 \mathrm{e}^{-\sqrt{\frac{\lambda}{6\pi}}t}\,, \tag{34}$$

for $n \neq 4$ and $n = 4$, respectively. Here, ϕ_0 is the initial value of the scalar field: $\phi_0 \equiv \phi(t_0) \geq \phi(t)$ for symmetric potentials $V(\phi) = V(-\phi)$, such that $V(\phi_0) \simeq M_\mathrm{P}^4$. For any n, the scale factor $a(t) = a_0\,(t/t_0)^{p(t)}$ can be written as a function of $\phi(t)$:

$$\frac{a(t)}{a_0} = \mathrm{e}^{\frac{2\pi}{nM_\mathrm{P}^2}\sqrt{2/3}\left[\phi_0^2 - \phi^2(t)\right]}\,. \tag{35}$$

During inflation the amplitudes of the quantum fluctuations are of the order of the Hubble parameter: $|\delta\phi| \simeq \frac{H}{2\pi}$ [10,20], so that

$$|\delta\phi| \simeq \frac{1}{2\pi\psi}\,; \tag{36}$$

$|\delta\phi|$ is dominated by the evolution of the fifth coordinate. Furthermore, inflation ends when the inflaton field assumes the value $\phi_e \simeq \frac{nM_\mathrm{P}}{4\sqrt{3\pi}}$. Since $H = \dot{a}/a$, from (13) and (35) we obtain the temporal dependence of the time-dependent power $p(t)$ [written as a function of $\phi(t)$]

$$p(t) = \frac{\frac{2\pi}{nM_\mathrm{P}^2}\sqrt{2/3}\left[\phi_0^2 - \phi^2(t)\right]}{\ln\left[\frac{t}{t_0}\right]}\,, \tag{37}$$

and, with more generality,

$$p(t) = \frac{N(t)}{\ln\left[\frac{t}{t_0}\right]}\,, \tag{38}$$

where $N(t) = \int_{t_0}^{t} H(t')\mathrm{d}t'$ is the number of e-folds from t_0 to t.

In order to illustrate the generality of the formalism here studied, in the next section we shall develop some particular inflationary examples described by symmetric ϕ-potentials.

4 Some examples

Slow-roll inflation is well described by symmetric potentials like (31) or $V(\phi) \sim \sinh^2(\beta\phi)$. In this section we shall study the dynamics of slow-roll inflation in quadratic, quartic and hyperbolic ϕ-potentials.

4.1 Massive scalar field

As a first example we consider a massive scalar field described by a quadratic potential $V(\phi) = \frac{m^2}{2}\phi^2$. This case corresponds to $\lambda = m^2$ and $n = 2$ in (31). Its temporal dependence is described by (33), so that

$$\phi(t) = \phi_0 - \frac{M_\mathrm{P} m}{\sqrt{2\pi}} t, \tag{39}$$

where m is the mass of the inflaton field. The Hubble parameter can be written as a function of ϕ:

$$H(\phi) \simeq 2\sqrt{\frac{\pi}{3}}\frac{m}{M_\mathrm{P}}\phi\,, \tag{40}$$

and $p(t)$ can be obtained from (38),

$$p(t) = \sqrt{\frac{2}{3}}\pi\frac{m}{M_\mathrm{P}}\frac{t}{\ln(t/t_0)}\left(\sqrt{\frac{2}{\pi}}\phi_0 - \frac{mM_\mathrm{P}}{2\pi}t\right)\,, \tag{41}$$

where $\phi_0 > \frac{1}{2}\sqrt{\frac{1}{2\pi}}mM_\mathrm{P}t_e$ (t_e is the time at the end of inflation), due to the fact that $p > 0$ in an expanding universe. In particular, $p > 1$ during the inflationary epoch. Note that p decreases with time during inflation. Thus, at the beginning p should take a value very large that decreases until values very close to $p \simeq 1$ at the end of inflation. The function u [see (26)], can be written as a function of the inflaton field,

$$u(\phi) = \frac{4\pi}{M_\mathrm{P}^2}\phi^2\,, \tag{42}$$

which is $u \gg 1$ because during inflation the slow-roll conditions imply

$$\phi^2 \gg \frac{M_\mathrm{P}^2}{4\pi}\,. \tag{43}$$

The velocities (27) can be written as a function of the inflaton field,

$$\begin{aligned} U^t &= \frac{2}{\sqrt{1 - \frac{M_\mathrm{P}^4}{16\pi^2\phi^4}}} \simeq 2\,, \\ U^R &= \frac{-2r}{\sqrt{\frac{16\pi^2\phi^4}{M_\mathrm{P}^4} - 1}} \simeq -2r\gamma(\phi)\,. \end{aligned} \tag{44}$$

The number of e-folds and the evolution of the fifth coordinate can be written as a function of the inflaton field,

$$N(\phi) = \frac{\pi}{2M_\mathrm{P}^2}\sqrt{\frac{2}{3}}\left(\phi_0^2 - \phi^2\right)\,, \tag{45}$$

$$\psi(\phi) = \frac{1}{2}\sqrt{\frac{3}{\pi}}\frac{M_\mathrm{P}}{m}\phi^{-1}\,, \tag{46}$$

where $\phi_0 = \frac{\sqrt{2}}{m}M_\mathrm{P}^2$ is the initial value of the inflaton field, $\frac{M_\mathrm{P}}{2\sqrt{3\pi}} < \phi < \phi_0$ and the initial value of the fifth coordinate $\psi_0 = H_0^{-1}$ is of the order of the Planck length:

$$\psi_0 \simeq \frac{1}{2}\sqrt{\frac{3}{2\pi}}\frac{1}{M_\mathrm{P}}\,. \tag{47}$$

4.2 Self-interacting scalar field

Another interesting example that describes a self-interacting scalar field is given by the quartic potential $V(\phi) = \frac{\lambda}{4}\phi^4$. Here, $\lambda \ll 1$ is a dimensionless constant and the classical evolution for the inflaton field is given by (34). The Hubble parameter is

$$H(\phi) \simeq \frac{1}{M_\mathrm{P}}\sqrt{\frac{2\pi\lambda}{3}}\phi^2\,, \tag{48}$$

and the temporal evolution for $\phi(t)$ is given by (34). Hence, the temporal dependence for $p(t)$ is

$$p(t) = \frac{\phi_0^2}{\sqrt{6}M_\mathrm{P}^2\ln(t/t_0)}\left(1 - \mathrm{e}^{-2\sqrt{\frac{\lambda}{6\pi}}t}\right)\,, \tag{49}$$

for $t > M_\mathrm{P}^{-1}$. Note that p decreases with time. The function u is given by

$$u(\phi) \simeq \frac{\phi^2}{M_\mathrm{P}^2} \gg 1\,, \tag{50}$$

such that $\phi^2 \gg M_{\rm P}^2$. The velocities (27) for a model $V(\phi) = \frac{\lambda}{4}\phi^4$ become

$$U^t = \frac{2}{\sqrt{1-\frac{M_{\rm P}^4}{\phi^4}}} \simeq 2\,, \qquad U^R = \frac{-2r}{\sqrt{\frac{\phi^4}{M_{\rm P}^4}-1}} \simeq -2r\gamma(\phi)\,. \tag{51}$$

On the other hand, the number of e-folds and the evolution of the fifth coordinate can be related to the inflaton field,

$$N(\phi) = \frac{\pi}{4M_{\rm P}^2}\sqrt{\frac{2}{3}}\left(\phi_0^2-\phi^2\right)\,, \tag{52}$$

$$\psi(\phi) = M_{\rm P}\sqrt{\frac{3}{2\pi\lambda}}\,\phi^{-2}\,, \tag{53}$$

where $\phi_0 = \left(\frac{4}{\lambda}\right)^{1/4} M_{\rm P} > \phi > \frac{M_{\rm P}}{\sqrt{3\pi}}$ and $\psi_0 \simeq \frac{1}{2}\sqrt{\frac{3}{2\pi}}\frac{1}{M_{\rm P}}$, which is exactly the value we found for a massive scalar field [see (47)].

4.3 Hyperbolic potential

As a third example we consider a symmetric potential given by

$$V(\phi) = V_0\,\sinh^2(\beta\phi)\,, \tag{54}$$

where the parameter $\beta > 0$ has dimensions of the inverse of the mass. The dynamics of the inflaton field is given by

$$\dot\phi \simeq -\frac{2V_0^{1/2}M_{\rm P}\beta}{\sqrt{24\pi}}\cosh(\beta\phi)\,, \tag{55}$$

so that the temporal evolution of the inflaton field yields

$$\phi(t) = \phi_0 - \ln\left\{\tan\left[-\frac{\sqrt{24\pi}}{\beta^2 M_{\rm P}^3}t\right]\right\}\,. \tag{56}$$

The Hubble parameter is

$$H(\phi) \simeq \sqrt{\frac{8\pi}{3}}\frac{V_0^{1/2}}{M_{\rm P}}\sinh(\beta\phi)\,, \tag{57}$$

so that the number of e-folds is

$$N[\phi(t)] = \frac{1}{2\beta}\left\{\ln\left[\tanh\left(\beta\phi(t)\right)-1\right] + \ln\left[\tanh\left(\beta\phi(t)\right)+1\right]\right\}_{\phi(t)}^{\phi_0}\,, \tag{58}$$

where $\phi(t)$ is given by (56). The slow-roll parameter γ [see (22)] for this model is

$$\gamma(\phi) \simeq \frac{M_{\rm P}^2}{4\pi}\beta^2\coth^2(\beta\phi)\,, \tag{59}$$

which complies with the slow-roll conditions for $\beta\phi_0 \simeq 1$ for $\beta \ll M_{\rm P}^{-1}$. Note that, as in the other examples, slow-roll conditions imply that ϕ must take trans-Planck values. Furthermore, the function $u \gg 1$ for this inflationary model is given by

$$u(\phi) \simeq \frac{4\pi}{M_{\rm P}^2\beta^2}\tanh^2(\beta\phi)\,. \tag{60}$$

Finally, the evolution of the fifth coordinate can be written as a function of ϕ:

$$\psi(\phi) \simeq \sqrt{\frac{3}{8\pi}}\frac{M_{\rm P}}{V_0^{1/2}}\mathrm{sech}(\beta\phi)\,, \tag{61}$$

so that the value of the fifth corrdinate in the metric (11) should be $\psi_0 \simeq \frac{1}{2}\sqrt{\frac{3}{2\pi}}\frac{1}{M_{\rm P}}\mathrm{sech}(\beta\phi_0) \lesssim \frac{1}{2}\sqrt{\frac{3}{2\pi}}\frac{1}{M_{\rm P}}$, which agree quite well with the value obtained in the other examples. To finalize, in this case the evolution of the power-law $p(t)$ for the scale factor, will be given by (38),

$$p(t) = \frac{1}{2\beta\ln(t/t_0)}\left\{\ln\left[\tanh\left(\beta\phi(t)\right)-1\right] + \ln\left[\tanh\left(\beta\phi(t)\right)+1\right]\right\}_{\phi(t)}^{\phi_0}\,, \tag{62}$$

which decreases during the inflationary stage.

5 Final comments

We have studied inflationary dynamics from non-compact KK theory. With this representation the universe can be viewed as born in a state with $S \simeq 0$ (i.e., in a 4D vacuum state $\mathbf{p} \simeq -\rho$), where the fifth coordinate is given by the Hubble horizon in a comoving frame $\mathrm{d}r = 0$, such that the effective 4D spacetime is a FRW metric:

$$\mathrm{d}S^2 = \mathrm{d}t^2 - \mathrm{e}^{2\int H(t)\mathrm{d}t}\mathrm{d}R^2 - \mathrm{d}L^2 \to \mathrm{d}s^2 = \mathrm{d}t^2 - \mathrm{e}^{2\int H(t)\mathrm{d}t}\mathrm{d}R^2\,.$$

In this frame, the effective 4D energy density and the pressure are

$$8\pi G\rho = 3H^2\,, \qquad 8\pi G\mathbf{p} = -(3H^2 + 2\dot H)\,.$$

In particular, slow-roll inflation was discussed for ϕ-symmetric polynomial (quadratic and quartic) and hyperbolic potentials. Note that the initial length ψ_0 we found corresponds to the primordial Hubble horizon H_0^{-1}. In all the cases here studied its value becomes below (but of the order of) the Planck length: $\psi_0 \lesssim \frac{1}{2}\sqrt{\frac{3}{2\pi}}\frac{1}{M_{\rm P}} \simeq 0.346\times10^{-34}$ m. Thus, $L \lesssim 0.346\times10^{-34}$ m should be the value of the spatial fifth coordinate in the metric (15). Another remarkable result of this paper resides in that the dynamics in slow-roll inflation is governed by the evolution of the fifth coordinate $\psi = H^{-1}$ through the geodesic $N = \arctan(1/u)$, such that the expression $\frac{3}{8\pi G\psi^2} \simeq V(\phi)$ is fulfilled. Furthermore, the quantum fluctuations are also given by ψ: $|\delta\phi| \simeq \frac{1}{2\pi\psi}$. Finally, the formalism could be extended to models with constant or variable cosmological parameters Λ. However, this topic would go beyond the scope of this paper.

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